



GREEDY ALGORITHM TO GENERATE CUTTING PATTERNS FOR CUTTING STOCK PROBLEM (1D AND 2D)

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Abstract

The efficient utilization of raw material is utmost important for the manufacturing plants in order to reduce material wastage and cost of production. Cutting stock problem (CSP) deals with cutting of small objects (finals), out of a larger one exhibits the problem of minimization of wastage of material and optimal utilization of space. The design of Pattern which is a unique combination of finals is a very tricky and have several well-established methods. However, there are no general and efficient method that is applicable to CSP since it is NP-hard problem. In this paper a novel hybrid approach to solve CSP has been develop and proposed. CSP problem has been decomposed in two parts viz. Pattern Generation and Optimization. Pattern Generation is done by Greedy Optimization and optimization is done by integer programming. This paper present solution for 1D-CSP and 2D-CSP using hybrid approach and deals with several case studies and sample problems to showcase the applicability of this hybrid solution. It is worthwhile to mention that this approach solves the CSP problem in minimum given time and presents quality solutions. CSP is decompose in two parts in which patterns generation is done by Greedy Algorithm and Optimization is done using Integer Programming. This novel approach to solve CSP problem shows that its performance is better in terms of quality of solution and time required to solve the problem viz-a-viz existing solutions available.

Keywords: Cutting stock problem, Raw, Final, Pattern, NP-hard problem, greedy algorithm, integer programming.

1 INTRODUCTION

The Cutting Stock Problem (CSP) involves cutting a set of base materials into smaller pieces (finals) to reduce the wastage of raw material, thereby increasing the productivity of the industry. Depending on the dimension of the raw CSP one dimension (1D), two dimension (2D) and three dimension (3D) cutting stock problem are classified. Raw material can be cut into finals in a number of ways, known as cutting patterns. The cutting pattern algorithm provides numerous combinations of raw-finals, out of which optimum patterns selected in CSP. Which will reduce the total waste by fulfilling the demand for the final. Generating cutting pattern is NP hard problem (Gilmore C., 1963) and very tricky part for CSP. This paper focuses on the cutting pattern generation technique for CSP. A greedy algorithm strategy is used to create a cutting pattern. The greedy algorithm is used to find the best possible patterns for each set of raw-finals, reducing the overall pattern generation time and its infinite combinations.

The first section of the paper presents a brief overview of the work in this field, divided by the field of research (depending on dimension of CSP such as one dimension, two dimension and tree dimension, different method used to solve CSP such as linear programming, dynamic programming, knapsack method and different heuristic method). The second section explains about to cutting stock models, In which the problem statement and mathematical structure are briefly explained with the pattern generation for 1D-CSP and 2D-CSP. The third and last section of the paper explain about cutting stock model, case study and computing experiments with a solution given by different solver. It summarizes various approaches adopted for the CSP.

1.1. Literature Review: CSP has a wide range of variations and practical solutions. The literature for CSP can be classified according to the type of Raw Stock (1D, 2D, 3D CSP), Solving Methods (Linear Programming, Heuristic Methods, Meteorological Methods and Hybrid Methods), Application Fields (Packaging, Aircraft and Tube Industry) Special constraints and Requirements. There are many practical solutions available for this CSP problem (Dykoff. H, 1981). A number of researchers have work on 1D, 2D CSPs but there is little literature available for 3D CSPs. 1D problem solving having attempted by linear programming approach (Gomori, 1961). He claims that possible cutting patterns are enhanced with the required cutting items and that the linear programming technique is not applicable to solving mathematical models with many variables.

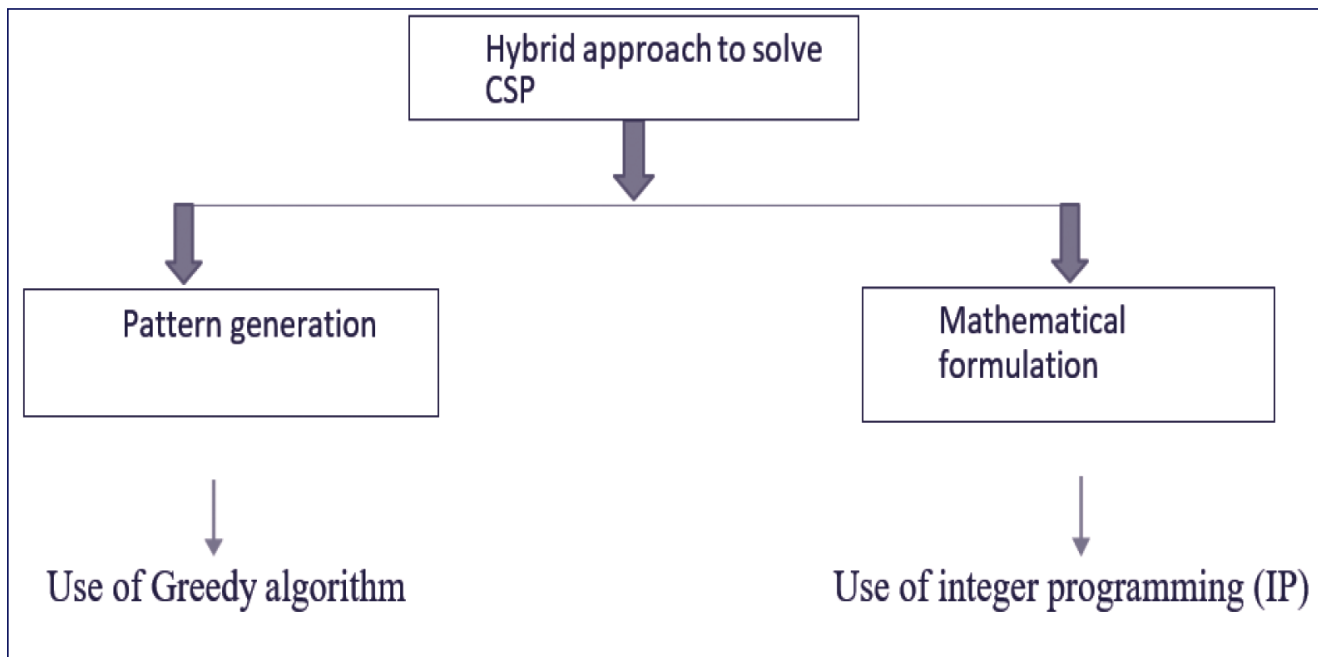
This can be solved by solving the number of incoming collocations in the lining programming formulation as a Knapask problem on the main step. Also they worked out a solution for two-dimensional cutting stock problem with the number of steps (2DCP) and the exponential model using the column generation technique (Gomori, 1965). (Whitlock, 1977) A three-dimensional rectangle-shaped cutting stock problem using tree-search algorithm for a dynamic programming process and node evaluation method for solving unstructured problem. Guillotine cuts and the maximum number of pieces from each item can be produced have been used as assumptions in this work. The application for the three dimensions consists of loading and scheduling containers with CSP limited resources. (J.Teich, 2001) One such application of reconfigurable computer chips.

There are numerous materials available for CSP for various reconciliation approaches. Morebito, 2008, have modeled CSP by a mixed integer linear formulation that aims to reduce the possibility of generating sufficiently sized residues to reduce material trim loss and reuse. Saad M., 2001, introduces a modified branch and bound algorithm and mathematical model to minimize total slip loss. The algorithm finds possible cutting patterns for CSP. Chen, 2011 a hybrid algorithm is used to solve 2D CSPs on multiple regular steel plates. In this paper they used a mathematical model with genetic mathematical rules. Mathematical models used to reduce the total aircraft time and genetic mathematical rules were applied to determine the layout order of the parts. Furrini, 2013, computed integer linear programming and branch-price technology have been compared for computation of 2D CSP. The paper describes the results of extensive computation experiments on examples of benchmarks set from the literature. Hassan Jawanashir, 2010 solved irregularly shaped pieces in rectangular two-dimension sheets of known dimensions using a simulated annealing process with the aim of reducing total cutting waste. Beasley has presented computational results for both uncontrolled 2D CSPs for stage cutting and guillotine cutting for both Heuristic and Optimal based dynamic programming. It indicates that the algorithm developed for staged cutting is more effective than guillotine cuts.

Classical CSP has numerous applications in different fields with different limitations and requirements. Jongsang L., 2013, have experimented with Knapsack-based heuristic algorithm for significant reduction of aging and cost in open die forging industry. Eggoun et al., 1993, introduced two new filtering methods that can be used in the context of complexes as well as for non-overlapping constraints for solving complex scheduling and placement problems.

As indicated by different authors, there is no common and efficient solution for cutting stock problems due to its complexity and tedious calculations. In this work, the attempt has been made to identify this issue pertaining to various aspects of the stock issue problems. It includes pattern creation techniques that will take care of all possible combinations and formulation for cutting raw and final shapes considering target warrant researcher. This work focuses on decomposition of the CSPs problem into two parts where pattern generation is attempted by greedy algorithm and there after optimization by integer programming. This is the hybrid solution for 1D CSP and 2D CSP and many case studies and sample problems have been demonstrated which prove the effectiveness and efficiency of this approach.

1.2. Methodology: Cutting stock problems (CSP) are combinatorial optimization problems, which occur in many real-world applications of business and industry, promoting many areas of research. Due to the complexity and pervasive nature of these problems, the literature contains many optimization formulations and solution approaches, according to their dimensions, application-aging and special constraints and requirements. Many researchers have worked on the cutting stock problem and developed different algorithms to solve the problem. Here we are providing a unique method for solving this classical CSP. Patterns are nothing but a cut of the raw to cut the finals so that the total waste can be minimized by meeting its demand. The pattern generation is challenging in itself, and it is having a great time. From available literature review method used for solving CSP pattern generation is part of either mathematical or heuristic algorithm only. In order to effectively generate patterns and improve overall solution time, we introduce a novel hybrid algorithm for CSP in which figure generation differs from mathematical programming Figure 1 gives an overview of the hybrid approach.



Figur 1: Hybrid methodology to solve CS

1.3 Paper Contribution: There is a numerous literature available for 1D and 2D CSPs that use combination integer programming, dynamic programming, heuristic, and metaheuristic methods, but there is no unique and accurate solution for CSP. This paper presents an innovative hybrid approach to solving CSP. Here, the CSP is divided into two parts. The first part contains mathematical design to reduce the overall cost of cutting and the second part uses a greedy algorithm to generate numerous patterns for cutting stocks and to select the optimal set of patterns to feed a mathematical formulation. This decomposition technique reduces overall time for solving CSP. This paper focuses on the greedy algorithm for cutting pattern generation and shows computationally how it compares well with other algorithms.

2 CUTTING STOCK MODELS

2.1. Mathematical Formulation: Optimization is nothing more than finding the best solution from a set of all feasible solutions that will satisfy every constrains. The optimum cutting stock problem is defined as cutting the main sheet into smaller pieces while reducing the total waste of the raw material or increasing the overall profit earned by cutting the smaller pieces from the main sheet. There are various optimization techniques available in which integer programming is used to solve CSP. CSP can be described as cutting down a number of smaller sets depending on the demand for the item, which is available in limited quantities, reducing total wastage.

❖ A verbal statement of the model is given:

Minimize: Total cost of raw (number of raw to be used by minimizing wastage)

Subject to:

1.Demand constraint: For all possible sizes of finals: the

number of finals produced from cutting raws according to the set of allowable cutting patterns must meet the demand.

2. Capacity constraint: For all raw: number of raw cut should be within the capacity of each raw.

The following integer program is a mathematical representation of the verbal model in the previous paragraph.

➤ Indices

p : cutting pattern

f : Finals

r : Raw

➤ Parameters

df : Demand for finals

$afpr$: Number of finals in cutting pattern

cr : cost of each raw

kr : Available capacity of each raw

➤ Variables

xpr : Number of raw cut with cutting pattern Minimize: $\sum p cr$
 xpr

Subject to:

$$\begin{array}{lll} \sum_{pr} afpr xpr & \geq & \forall f \\ \sum_{pr} xpr & \leq & kr \quad \forall r \\ xpr & \geq 0 & in \forall p, r \end{array}$$

2.2. 1-D CSP Pattern Generation: A cutting pattern is a specific procedure for each size of final object of how many piece are cut from one raw material. Of course, there are many ways. For small examples, the size of a set of cutting patterns may not be excessive, but in most real problems the number of possible cutting patterns can be in the millions.

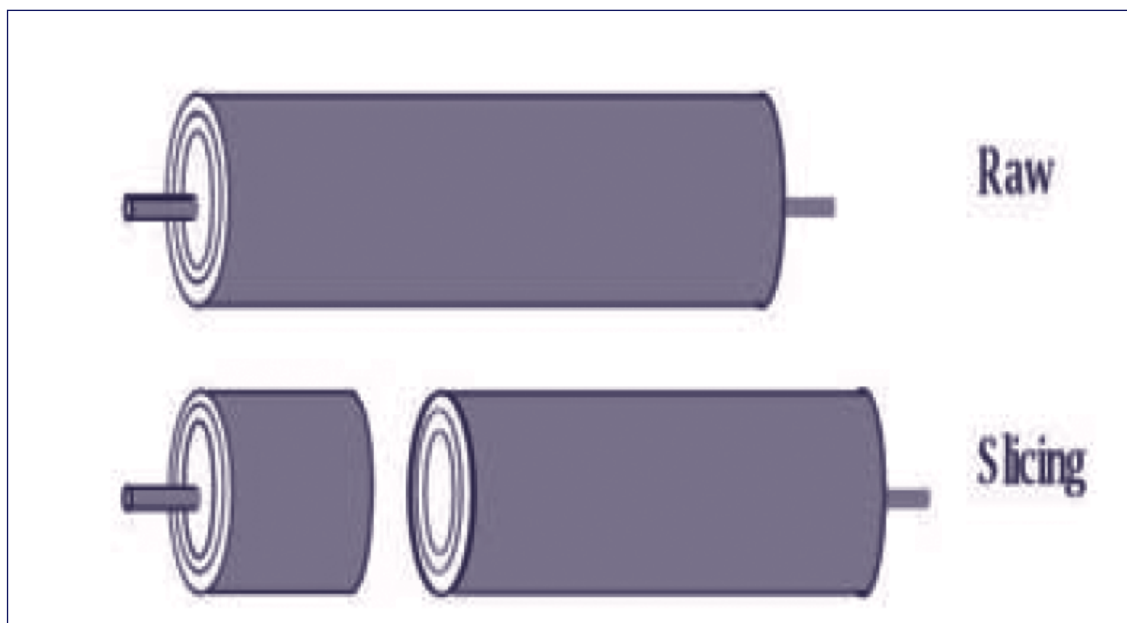


Figure 2: 1D pattern

Algorithm for 1D-CSP is explained in following sections using flowchart:

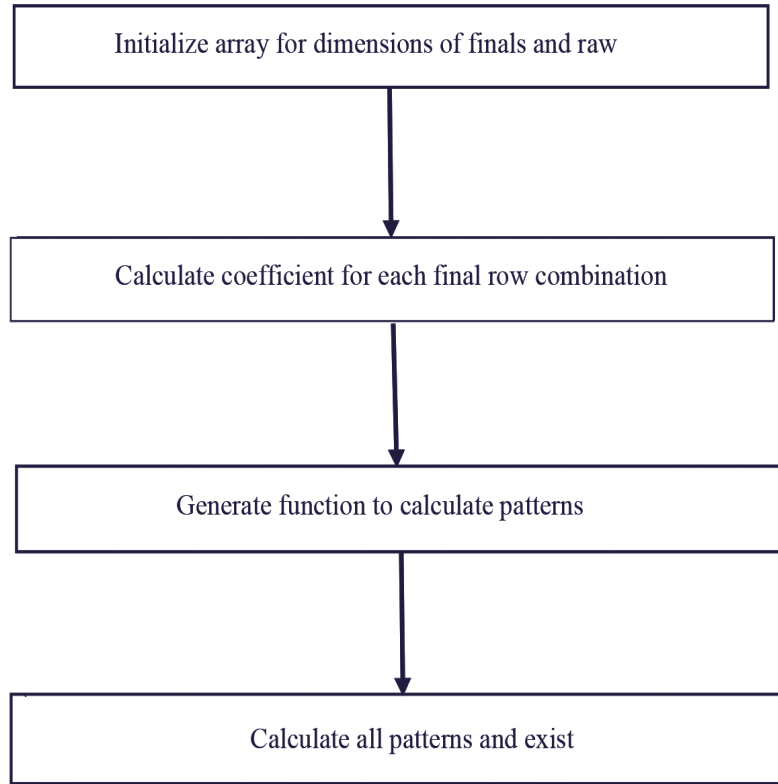


Figure 3: Flowchart for 1D pattern generation

Initialize array for finals and raw dimensions: There are different lengths of raw materials for each application and for different customers. As well as generate the pattern along with the final length and demand, this information is required at all times. This information is updated every time in micro-enabled Excel that has a code for pattern generation. For example, here are two raw materials with lengths 20 and 40 m, respectively, R1 and R2, and four finals 5, 9, 10, 4 m length, respectively F1, F2, F3, F4.

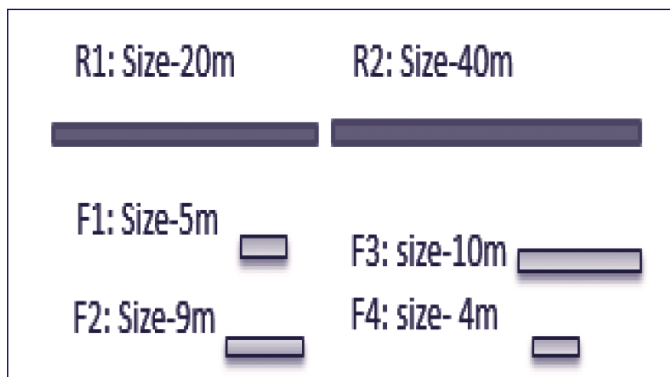


Figure 4: step 1 (1D)

Calculate the coefficient for each final raw combination: The next step in the algorithm is to calculate the maximum coefficient for each final raw combination. These coefficients tell us how many times the final match can occur in each raw material.

$$\text{Maximum coefficient} = \frac{\text{raw length}}{\text{final length}}$$

Generate a function to calculate the pattern: After calculating the maximum coefficient for the combination of the individual final-raw combination, we need to consider each of the components to determine who can generate the final combination pattern. The constraint to finalizing the final combination from raw are as follows.

$$\text{length of raw} \leq \sum \text{final coefficient} * \text{final1} + \text{coefficient} * \text{final2} + \text{coefficient} * \text{final3} + \text{coefficient} * \text{final4} \\ \forall \text{ rows}$$

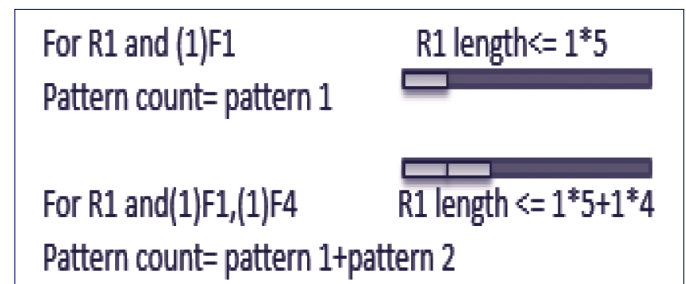


Figure 5: Step 3 (1D)

Calculate all patterns and existing models: The last step of this algorithm is to calculate all possible combinations of final and raw materials; all patterns can draw. After doing that the algorithm comes into existence.

2.3. 2-D CSP Pattern Generation: Pattern generation technology is the most difficult part for 2D-CSP and is different

from 1D-CSPs in terms of additional dimension, possible shapes, orientation of shape and non-overlapping criteria.

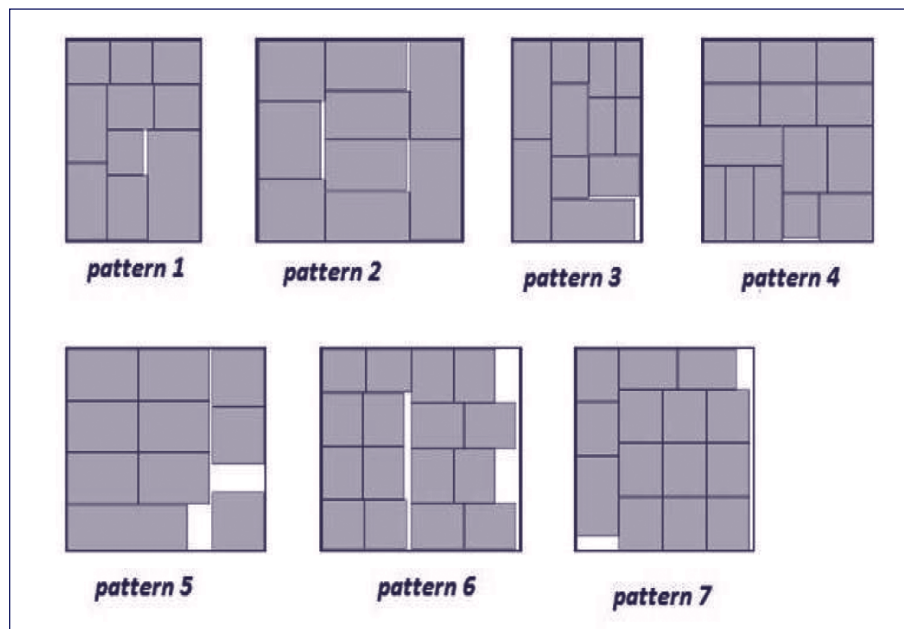


Figure 6: Pattern generation 2D

There are some assumption while generation patterns for 2D CSP:-

1. Only square and rectangular shapes of raw and finals are allowed.
2. Only 90 degree rotation is allowed for finals.

Pattern generation technology is the most difficult part for 2D-CSPs and is different from 1D-CSPs in terms of additional dimension, potential shapes, shape approaches and non-overlapping criteria. A flowchart of the greedy algorithm for 2D-CSP is presented in Figure 8.

Coordinate geometry (X,Y): In 1D we deal with only one dimension of raw and final, which can be either length or width or height. But 2D is a combination of length width or length height, or height width of both final and raw because in our estimation, only rectangle or square shape is likely, so only these two dimensions are possible. So we have to consider coordinate geometry when creating patterns. The coordinates and the finals in the coordinated geometry are presented on the XY plane such that each axis represents one of the dimensions shown in Figure 6.

Different shapes and elements of finals: In practical scenario there is the possibility of the final and various shapes of elements such as rectangles, squares, circles, triangles, contours or any other irregular shapes. If we consider these shapes then the finals and the raw may have a wider combination. Our research scope is final and limited to raw rectangles and square shapes.

The final direction is the probability of placing finals on every degree and hence one more possibility of pattern. An infinite

pattern will arise if we consider the other factor as well as each degree for each final. So to limit the pattern creation in our algorithms we have considered only the 90 degree rotation of the finals.

Non-overlapping criteria: When placing one end over the other, the basic criteria should not overlap one over the other. That pattern is useless except that it overlaps on one another. This is why our algorithm has created special non-overlapping criteria to handle this situation. A large number of instances will be generated by the greedy algorithm taking into account only the above factors, and they are filtered out only at that stage. This filter pattern is stored in the Excel database, which is the feed for integer programming in CPLEX, minimizing overall time.

3. COMPUTATION EXPERIMENTS

3.1 Case Study and Solution for 1D-CSP: We compare our hybrid solution result for 1D-CSP, with the built-in column generation technology in CPLX and AIMS solver. AIMMS provides a comprehensive development environment for creating high performance decision support and advanced planning applications to optimize strategic operations. The CPLEX Solver from IBM ILOG is a high performance solver for linear programming (LP), mixed integer programming (MIP) and quadratic programming (QP/QCP/MIQP/MIQCP) problems. 1D-CSP can be stated because the customer has a limited availability bar stock (raw) with a certain length, which requires a cut in the final piece to be applied for a certain amount (demand). What is economically way, in that it can cut bar stock to reduce total waste economically? The data needed for this problem is given in the following section.

Table 1: Input parameter for 1D-CSP

Sr No.	Raw size (in cm)	Cost per raw (in Rs)
1	1000	1

Sr No.	Final size (in cm)	Demand for final
1	450	200
2	350	531
3	310	395
4	267	150
5	220	157
6	201	67
7	189	300
8	167	500
9	140	211
10	100	321
11	50	20

Figure 7: flowchart for 2D pattern generation

We introduced the solution for 1D-CSP in CPLEX and AIMMS through the column generated technology, even though hybrid approaches that first create patterns through the greedy algorithm and solve the optimization problem by integer

programming. The results are presented in Table 2 The results directly show that the total cost and total solutions time in the hybrid approach are minimal.

Table 2: Result comparison for 1D

Solver	Result (Total cost in Rs)	Solution time in sec.
CPLEX (in built column generation technique)	1433	15
AIMMS (in built column generation technique)	629	13
CPLEX (hybrid approach)	250	5

3.2 Case study and solution for 2D-CSP

2D-CSP have application in area such as glass, leather, aircraft, sheet metal industry. 1D- Solutions ready for CSP (by using column generation technique) are present in CPLEX and AIMMS, but there is no such solution for 2D-CSP. To explain hybrid method i.e. pattern generation by greedy algorithm and optimization by integer programming, we illustrate here a real-life example. consider a real-life problem in which a customer

has a sheet metal (raw) of 20 meters in length and a height of 40 meters, priced at Rs 400, with a maximum availability of 200 pieces. These raw sheets are cut from metal to the small piece of sheet (final) as per customer's requirement. The size of the smaller sheets and their demand are given in the table. So how do I cut a long raw sheet (pattern) so that it can be economically complete with small wire requirement and minimum wastage?

Input from customer

Table 3: Raw information (2D)

Sr. no	Raw size (in meters)	Availability	Cost per raw
1	20 x 40	200	100

Table 4: Final information (2D)

Sr. no	Final size (in meters)	Demand for finals
1	5 x 10	150
2	4 x 5	200

We have introduced a solution for 2D-CSP in CPLEX through a hybrid approach that first creates a pattern through a greedy algorithm and solves the optimization problem through integer

programming. Results are presented in Table 5. There is no ready solution or method available for solving 2D-CSP. Hence we cannot compare this result with any other.

Table 5: CPLEX final result (2D)

Solver	Result (total cost in Rs)	Solution time in sec.
CPLEX (hybrid approach)	5000	10

4. CONCLUSION

In this work attempt has been made to solve the CSP for 1D and 2D and its effectiveness have been compared with column generation techniques of CPLEX and AIMMS. This paper demonstrates that optimization through integer programming and hybrid approach using greedy algorithms performs better in terms of time and quality solutions and this hybrid approach is best suited for CSP. It shows that the maximum utilization area can be filtered out from results by separating pattern generation technique and attempting it by Greedy Algorithm. It provides a superior solution than column generation technique. Pattern generation is separated from mathematical formulation, hence mathematical formulation becomes generic for 1D, 2D, 3D CSP. This implies in quick and practical implementation and robustness of formulation. To apply 1D, 2D CSP hybrid solution in practical scenario different customer requirements easily adapted through Greedy Algorithm without changing mathematical formulation which ensure optimal solution each time.

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